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Using a Monte Carlo Approach to Determine the Radioxenon Probability Density Function at an IMS Station for Background Estimation

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Introduction

- It is known that radioxenon released from nuclear facilities such as medical isotope production facilities (MIPF) and nuclear power plants (NPP) influences International Monitoring System (IMS) radionuclide stations*[1], *[2].

- For the purposes of monitoring nuclear explosions, it is important to better understand the radioxenon background based on these nuclear facilities.

- We investigate a methodology of estimation of probability density functions (PDF) of activity concentration at IMS radionuclide stations using a Monte Carlo approach*[3], based on the emission from known nuclear facilities and source receptor sensitivity (SRS) data.

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*[1] See Kuśmierczyk-Michulec et al., 2022. Characterisation of Xe-133 background at the IMS stations in the East Asian region: Insights based on known sources and atmospheric transport modelling. J. Environ. Radioact. 255 (2022) 107033.

*[2] See Kalinowski, 2023. Global emission inventory of 131mXe, 133Xe, 133mXe, and 135Xe from all kinds of nuclear facilities for the reference year 2014. J. Environ. Radioact. 261 (2023) 107121.

*[3] See JCGM 101:2008, 2008. Evaluation of measurement data – Supplement 1 to the "Guide to the expression of uncertainty in measurement" – Propagation of distributions using a Monte Carlo method.

Probability density function of background concentration

$$C = \sum_{k=1}^{n} M_k S_k$$

*1

C: activity concentration at receptor [Bq/m^3]
M: source receptor sensitivity (SRS) [m^{-3}]
S : emission amount at source [Bq]
k : time interval

Expression using probability density functions (PDF) *2, *3

$$p_C(c) = \int_{C_1}\int_{C_{1,2}} \cdots \int_{C_{1,2,\dots,n-1}} dc_1\, dc_{1,2} \dots dc_{1,2,\dots,n-1} \left\{\int_{M_1} p_{M_1}(m_k)\, p_{S_1}\!\left(\frac{c_1}{m_1}\right)\frac{1}{|m_1|}\, dm_1\right\}$$
$$\left\{\int_{M_2} p_{M_2}(m_2)\, p_{S_2}\!\left(\frac{c_{1,2}-c_1}{m_2}\right)\frac{1}{|m_2|}\, dm_2\right\} \dots \left\{\int_{M_n} p_{M_n}(m_n)\, p_{S_n}\!\left(\frac{c_{1,2,\dots,n}-c_{1,2,\dots,n-1}}{m_n}\right)\frac{1}{|m_n|}\, dm_n\right\}$$

$p_C(c)$: concentration PDF
$p_M(m)$: SRS PDF
$p_S(s)$: emission PDF
$C_{1,2,\dots,n} \equiv C_1 + C_2 + \cdots + C_n = C$

- The Monte Carlo method (MCM) is used to solve the equation numerically.

*1 See Wotawa et al., 2003. Atmospheric transport modelling in support of CTBT verification: Overview and basic concepts. Atmos. Environ. 37, 2529 – 2537.

*2 See Glen et al., 2004. Computing the distribution of the product of two continuous random variables. Computational Statistic & Data Analysis, 44(3), 451–464.

*3 See Mallick et al., 2018. A Note on Sum, Difference, Product and Ratio of Kumaraswamy Random Variables. Mathematics, Statistics and Computer Science Faculty Research and Publications, 647.

Comparative evaluation between background and measurement

1. Relative probability of hypothesis

Bayes' theorem

$$P(S_i|A) = \frac{P(S_i)\,P(A|S_i)}{\sum_{j=1}^{N} P(S_j)\,P(A|S_j)} \quad (1 \le i \le N)$$

where

S_i: signals (emission sources)
A: event which gets a measured value with confidence interval $[c_M - 2\sigma, c_M + 2\sigma]$ (where c_M is the measured activity concentration [Bq/m^3], σ is the standard deviation of measurement [Bq/m^3])
$P(S_i|A)$: probability of S_i given A
$erf(\varepsilon)$: error function

$$\frac{P(S_i|A)}{P(S_1|A)} = \frac{P(S_i)\,P(A|S_i)}{P(S_1)\,P(A|S_1)} = \frac{P(S_i)\left[erf\left(\frac{c_M + 2\sigma - c_{S_i}}{\sqrt{2}\sigma}\right) - erf\left(\frac{c_M - 2\sigma - c_{S_i}}{\sqrt{2}\sigma}\right)\right]}{P(S_1)\left[erf\left(\frac{c_M + 2\sigma - c_{S_1}}{\sqrt{2}\sigma}\right) - erf\left(\frac{c_M - 2\sigma - c_{S_1}}{\sqrt{2}\sigma}\right)\right]}$$

- Realistically, it is difficult to know all of emission sources that influence IMS radionuclide stations.
- Using the ratio of $P(S_i|A)$ to $P(S_1|A)$ can clear the denominator.

Comparative evaluation between background and measurement

2. Residual approach

$$C_D \equiv C_M - C_B$$

C_M: activity concentration of measurement [Bq/m^3]
C_B: activity concentration of background [Bq/m^3]
C_D: residual between measurement and background [Bq/m^3]

Expression using PDF and Bayesian approach *[1] *[2]

$$p_{\tilde{D}}(\tilde{d}|d) = l_D p_{\tilde{D}}(\tilde{d}) \int_{c_B} p_M(c_D + c_B) p_B(c_B)\, dc_B$$

$p_M(c_M)$: measurement PDF
$p_B(c_B)$: background PDF
l_D: normalization constant

$$p_{\tilde{D}}(\tilde{d}) = \begin{cases} 1 & if\ \tilde{d} \geq 0 \\ 0 & otherwise \end{cases}$$

- MCM is used to solve the equation numerically.

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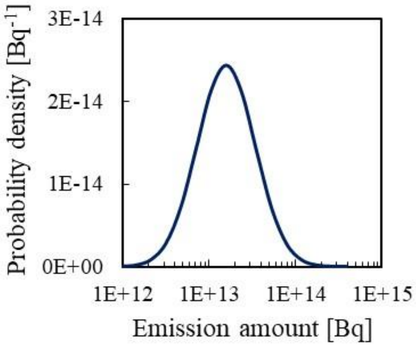
*[1] See Mallick et al., 2018. A Note on Sum, Difference, Product and Ratio of Kumaraswamy Random Variables. Mathematics, Statistics and Computer Science Faculty Research and Publications, 647.

*[2] See ISO 11929-2:2019(E), 2019. Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval for measurements of ionizing radiation – Fundamentals and application – Part 2: Advanced applications.

Prior assumptions

- Emission data from Canadian Nuclear Laboratories (CNL) provided by CNL*[1], *[2] are used.
- Emission amount from CNL is assumed to follow the log-normal distribution, since the emission amount usually varies hour to hour greatly*[1].
- SRS value is assumed to follow separate delta distribution per SRS value.
- Measured activity can be assumed to follow the Gaussian distribution*[4]. Under assumption of constant activity concentration profile, the measured activity concentration is assumed to follow the Gaussian distribution.



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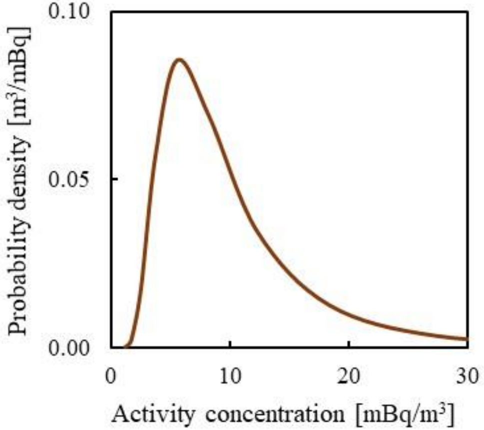
*[1] See Maurer et al., 2022. Third international challenge to model the medium- to long-range transport of radioxenon to four Comprehensive Nuclear-Test-Ban Treaty monitoring stations. J. Environ. Radioact. 255 (2022) 106968.

*[2] See Kalinowski, 2023. Global emission inventory of 131mXe, 133Xe, 133mXe, and 135Xe from all kinds of nuclear facilities for the reference year 2014. J. Environ. Radioact. 261 (2023) 107121.

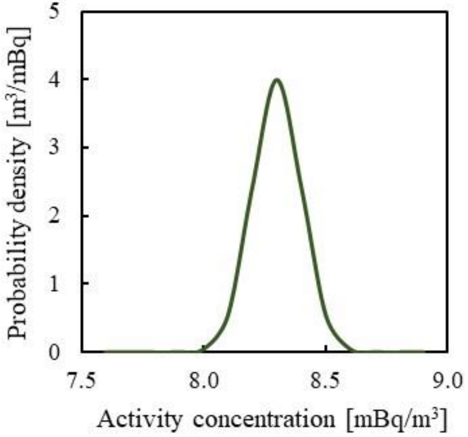
*[3] See ISO 11929-1:2019(E), 2019. Determination of the characteristic limits (decision threshold, detection limit and limits of thecoverage interval for measurements of ionizing radiation – Fundamentals and application – Part 1: Elementary applications.

Case study 1
Xe-133 at CAX17 on 3 March 2014

Background estimated vs. real measurement



Background estimated

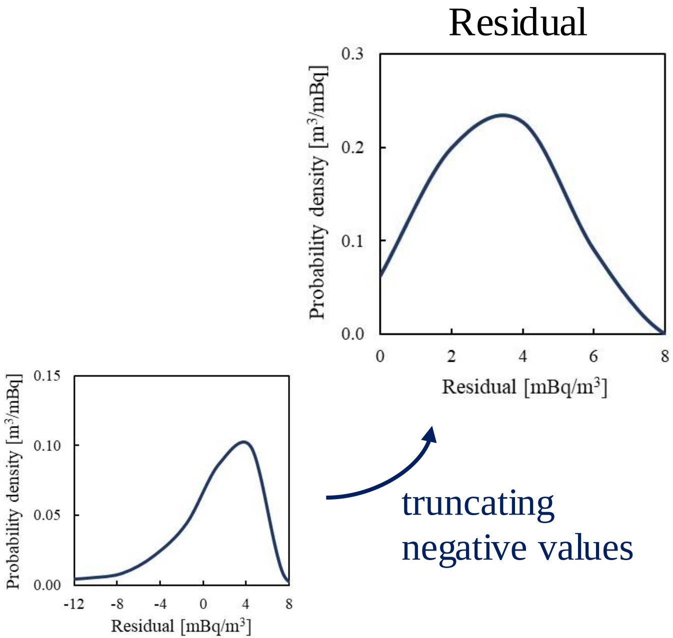


Measurement

- The mean value of the background for CAX17 (in St. John’s, Canada) on 3 March 2014 is 7.3 mBq/m^3. On the other hand, the measured activity concentration is 8.3 mBq/m^3.

Case study 1
Xe-133 at CAX17 on 3 March 2014

Residual approach

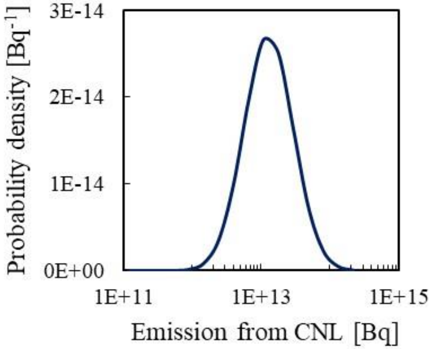


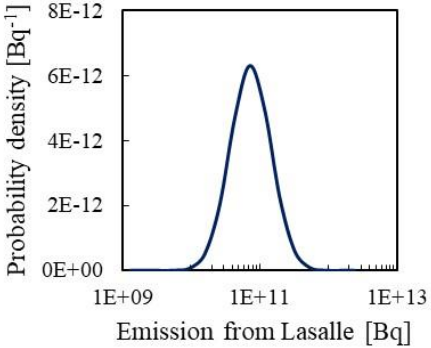
- Residual PDF becomes truncated (zero for negative values) distribution, since the true value of residual (= measurement − background) cannot be negative.
- The p-value is 0.08. When we select the threshold of p-value is 0.05, there is no statistically significant difference between the measurement and the background.
- It can be said that the Xe-133 at CAX17 on 3 March 2014 is originated from CNL.

Case study 2
Xe-133 at USX75 at 23:00 on 26 January 2014

Prior assumptions

- Emission data from Canadian Nuclear Laboratories (CNL) and Lasalle NPP in the USA [*1, *2, *3] are used.
- Emission amount from CNL and Lasalle is assumed to follow the log-normal distribution, since the emission amount usually varies hour to hour greatly[*1].
- SRS value is assumed to follow separate delta distribution per SRS value.
- Measured activity can be assumed to follow the Gaussian distribution[*4]. Under assumption of constant activity concentration profile, the measured activity concentration is assumed to follow the Gaussian distribution.





[*1] See Kalinowski et al., 2009. Global radioxenon emission inventory based on nuclear power reactor reports. J. Environ. Radioact. 100 (2009) 58–70.

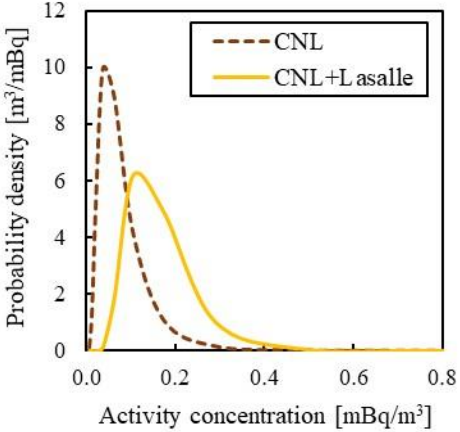
[*2] See Maurer et al., 2022. Third international challenge to model the medium- to long-range transport of radioxenon to four Comprehensive Nuclear-Test-Ban Treaty monitoring stations. J. Environ. Radioact. 255 (2022) 106968.

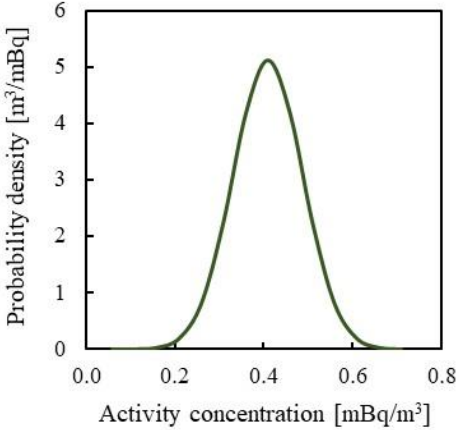
[*3] See Kalinowski, 2023. Global emission inventory of 131mXe, 133Xe, 133mXe, and 135Xe from all kinds of nuclear facilities for the reference year 2014. J. Environ. Radioact. 261 (2023) 107121.

[*4] See ISO 11929-1:2019(E), 2019. Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval for measurements of ionizing radiation – Fundamentals and application – Part 1: Elementary applications.

Case study 2
Xe-133 at USX75 at 23:00 on 26 January 2014

Background estimated vs. real measurement


Background estimated

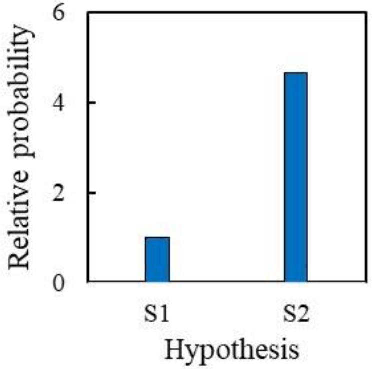

Measurement

- The mean value of the background only CNL for USX75 (in Charlottesville, USA) at 23:00 on 26 January 2014 is 0.06 mBq/m^3 and that of both CNL and Lasalle is 0.14 mBq/m^3.
- On the other hand, the measured activity concentration is 0.41 mBq/m^3.

Case study 2
Xe-133 at USX75 at 23:00 on 26 January 2014

Relative probability of hypothesis

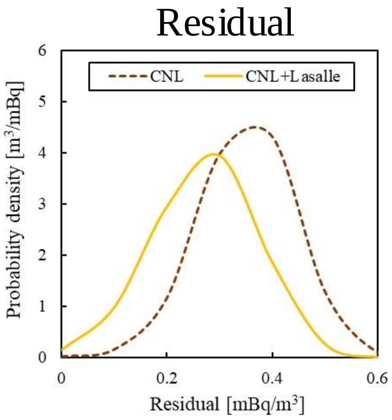
Relative probability vs. hypothesis

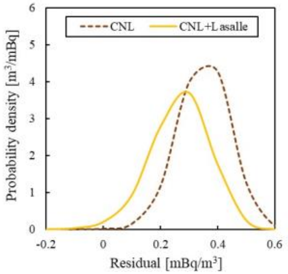


- S1 indicates a hypothesis that the emission source of Xe-133 at USX75 at 23:00 on 26 January 2014 is only CNL. S2 indicates a hypothesis that the emission sources of Xe-133 at USX75 at 23:00 on 26 January 2014 are CNL and Lasalle.
- It can be said that the probability that the emission source of the Xe-133 is CNL and Lasalle is higher than the probability that the emission source is only CNL.

Case study 2
Xe-133 at USX75 at 23:00 on 26 January 2014

Residual approach





truncating
negative values

- Residual PDF becomes truncated (zero for negative values) distribution, since the true value of residual (= measurement − background) cannot be negative.
- The p-value of CNL is 0.0005. When we select the threshold of p-value is 0.05, there is a statistically significant difference between the background (when we assume the emission source is only CNL) and the measurement.
- The p-value of CNL and Lasalle NPP is 0.01, much larger than that of only CNL. But there is still a statistically significant difference between the background (when we assume the emission sources are CNL and Lasalle) and the measurement.
- It can be said that there are other nuclear facilities that influenced USX75 at 23:00 on 26 January 2014 as well as CNL and Lasalle.

Conclusions

- The Monte Carlo approach described here could be one of several prospective approaches to predict the activity concentrations of CTBT-relevant radioxenons at IMS radionuclide stations in the prototype xenon background estimation tool (XeBET) software*[1]. It can also be used in characterization of CTBT-relevant nuclear events for expert technical analysis (ETA)*[2].

- Regarding Xe-133 at CAX17 on 3 March 2014, there is no statistically significant difference between measurement and background. It might be possible that the emission source is CNL.

- Regarding Xe-133 at USX75 at 23:00 on 26 January 2014, there is a statistically significant difference between measurement and background, when we assume the emission sources are CNL and Lasalle. It might be possible that the emission sources are not only CNL and Lasalle but also the other nuclear facilities.

*[1] See Schoemaker et al., 2023. Supporting a Better Screening for CTBT-relevant Events against a Radioxenon Background: XeBET Research and Development. Proceedings of the INMM & ESARDA Joint Annual Meeting, May 22-26, 2023.

*[2] See Liu et al., 2023. Characterization of CTBT-Relevant Radioxenon Detections at IMS Stations Using Isotopic Activity Ratio Analysis. Pure and Applied Geophysics, 180, 1521–1540.

Future works

- Further utilization of the Source Term Analysis of Xenon (STAX) data*[1] and investigation on emission released from nuclear facilities, for better estimation of the emission PDF.

- Investigation on *e.g.* ensemble approaches*[2] and high-resolution ATM (HRATM)*[3], for better estimation of the SRS PDF.

- Enhancement of *e.g.* alternative beta-gamma analysis method (ABGAM)*[4], for better estimation of the measurement PDF.

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*[1] See Eslinger et al., 2022. Using STAX data to predict IMS radioxenon concentrations. Journal of Environmental Radioactivity 250 (2022) 106916.

*[2] See Generoso et al., 2023. Addressing the quantification of meteorological uncertainties in the atmospheric transport simulations of the 133Xe industrial background. J. Environ. Radioact. 270 (2023) 107263.

*[3] See Tipka et al., 2022. Investigating the potential benefits of high-resolution ATM to the possible source localization in complex terrain. AGU Fall Meeting 2022, held in Chicago, IL, 12-16 December 2022, id. A52O-1180.

*[4] See Liu et al., 2023. Characterization of CTBT-relevant nuclear events using Isotopic Activity Ratios and Requirements on Spectrum Analysis. Expert Meetings on Special Studies and Expert Technical Analysis with Waveform Methods and with Radionuclide and Atmospheric Transport Modelling Methods, held in Daejeon, Republic of Korea, 16 – 20 October 2023.

Acknowledgments

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Thank you !

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